The η/*s* ratio in finite nuclei

N. Auerbach[1] and S. Shlomo

[1]*School of Physics and Astronomy, Tel Aviv University Tel Aviv, 69978, Israel*

In certain supersymmetric gauge theories one finds [1] that the ratio of shear viscosity η to entropy density *s* is equal to:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \approx 6.05\times 10^{-13}\,Ks \tag{1}$$

Where k_B is the Boltzmann constant. It has been conjectured that this ratio is the lower limit for a large class of quantum field theories. The analysis of the ultrarelativistic heavy ion collisions data from RHIC leads to [2]

$$\frac{\eta}{s} \le 5\times \frac{\hbar}{4\pi k_B} \tag{2}$$

It seems to indicate that the state of matter produced behaves like a liquid with the above ratio being close to the lower limit [2]. Thus the matter produced behaves as a perfect fluid. In this work [3] we first demonstrate that a consistent value for η is deduced from (i) analysis of the width of giant resonances within the hydrodynamic model, (ii) kinetic theory, and from (iii) the process of fission described using liquid drop models. We then provide a simple assessment of the entropy density.

Most of the giant resonances, at excitation energies in the range of 10-40 MeV have a finite life time and carry a width. Following the success of the hydrodynamical models an attempt was made to link the widths of these resonances to the viscosity of the proton-neutron fluids [4]. In Ref. [4] a set of coupled hydrodynamical equations of the Navier- Stokes type was used to describe the flow of two viscous fluids, of protons and neutrons. Solving these equations with appropriate boundary conditions and for various multipolarities one obtains eigenvalues containing a real and imaginary part. The real part represents the energy of the excitation and the imaginary part depends represents the lifetime of the excitation. The mass dependence A of the computed widths exhibits the experimental trends of the giant isoscalar resonances. As a result of such calculation one obtains a value for the shear viscosity is:

$$\eta \approx 1\times 10^{-23}\ \text{MeVfm}^{-3}\ \text{sec} \tag{3}$$

Recently, in Ref. [5], the authors described the dynamics of cold and hot nuclei within a generalized Fermi liquid drop model by employing a collision kinetic equation, which properly accounts for the dissipative propagation of sound waves in finite nuclei and nuclear matter. For a temperature $T<\varepsilon_F$ and excitation energy $\hbar\omega < \varepsilon_F$ of the sound wave, one finds for the collision viscosity

$$\eta = \frac{2}{5}\rho\varepsilon_F \frac{\tau_{coll}}{1+(\omega\tau_{coll})^2}, \quad \tau_{coll} = \frac{\tau_0}{1+(\hbar\omega/2\pi T)^2}, \quad \tau_0 = \hbar\alpha/T^2 \tag{4}$$

In Eq (4), τ_{coll} is the Landau ansatz for the collision relaxation time deducted from the collision integral. The value of α is sensitive to the in-medium-nucleon-nucleon scattering cross section. Taking the in-medium cross-section to be ½ of the free nucleon-nucleon cross section, one finds [5] that α=9.2 MeV. In Fig. 1a the value of η as a function of the temperature T, obtained using the values of ε_F = 40 MeV, ρ = 0.16 fm^{-3}, α = 9.2 MeV and $\hbar\omega$ = 20 MeV. Note that our results are relevant for temperature 0.5 MeV < T < 5 MeV, were giant resonances exist.

At low temperature, T << ε_F, the relation between the thermal excitation energy E^* and T is given by

$$E^* = aT^2 \tag{5}$$

where a is the level density parameter. For an ideal Fermi gas, for A=200 and a thermal excitation energy $E^* \approx \hbar\omega$ = 20 MeV, one obtains $T \approx 1$ MeV which then, using Eqs. (4-6), gives:

$$\eta \approx 0.5\times10^{-23} \text{ MeV fm}^{-3} \text{ sec} \tag{6}$$

Another type of collective motion encountered in the dynamics of nuclei is the process of fission. A number of works appeared in the literature which dealt with the dynamics of fission in heavy nuclei using viscous liquid drop models. For example, in [6] the authors use a macroscopic approach and solve classical equations of motion for the fissioning nucleus. They apply this to spontaneous and induced fission. They find that the average value for the shear viscosity that reproduces best the data is:

$$\eta \approx (0.9 \pm 0.3)\times10^{-23} \text{ MeV fm}^{-3} \text{ sec} \tag{7}$$

For low temperature, T < ε_F, the entropy density s of a nucleus is given in terms of the entropy S by the simple expression [5]:

$$s = \frac{\rho}{A} S, \qquad S=2aT, \tag{8}$$

In Fig. 1c we show η/s as a function of temperature. The curve has the characteristic behavior found in other fluids [1], the minimum at $T = 3$ MeV, with the value of $\eta/s \approx 3 \times \hbar/4\pi k_B$.

We therefore have for a nucleus the value [3]:

$$\frac{\eta}{s} \approx (2-12)\times\frac{\hbar}{4\pi k_B} \tag{9}$$

Comparing our results for finite nuclei at low temperature to that of RHIC, Eq. (2), we see that the deduced ratio (especially the lower limit) is not drastically different from the RHIC result. More studies are required in order to understand this point.

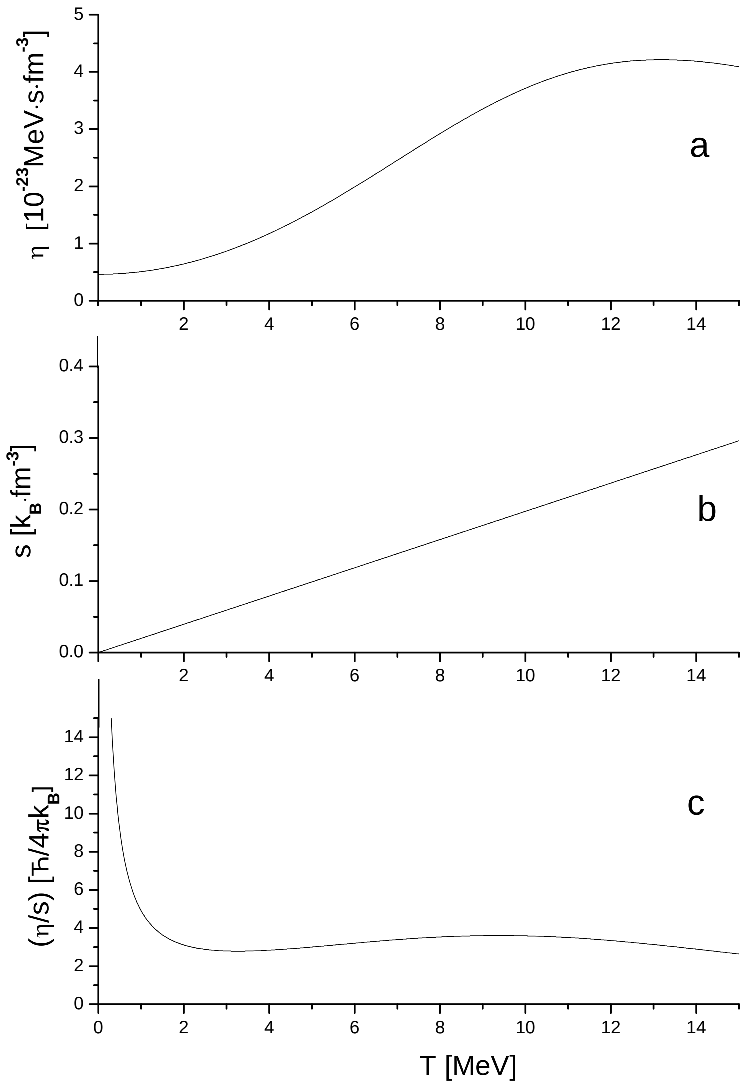

FIG. 1. The nuclear shear viscosity η (a), entropy density *s* (b) and their ratio, (η/*s*) (c), in units of $\hbar/4\pi k_B$, as functions of the temperature, T. The values of the parameters used in the calculations are; $\varepsilon_F = 40$ MeV, $\rho = 0.16$ fm^{-3}, $\alpha = 9.2$ MeV and $\hbar\omega = 20$ MeV.

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